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Transient Analysis of Plate Coupled with Fluid

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This paper deals with finite element modelling of three-dimensional fluid-structure interaction problem. Mathematical model for the structure is developed using a first order shell deformation theory and fluid is assumed to be acoustically compressible and wave equation is used to model it. Displacement of the nodes are the structural variable and velocity potential is taken as the fluid variable, the fluid is always in contact with the structure i.e. no separation. The interface conditions will give the coupling between the fluid and the structure. The changes in dynamic response of the structure such as resonant frequency due to added mass effect of fluid media is evaluated. A Finite element program in MATLAB developed to predict the dynamic response of the same structure including fluid structure interaction. Transient analysis conducted using ANSYS CFX for the fluid structure interaction problem of a vibrating plate attached to a box containing fluid.

Keywords—vibration;damping;resonance;transient analysis;viscosity;density

I. INTRODUCTION

Fluid-structure interaction is a common phenomenon in aerospace, marine, mechanical and civil engineering field. In fluid-structure interaction (FSI), at the interface, a property of the fluid influences a property of the solid and, crucially, vice versa. Finite Element Method (FEM) has traditionally been used as numerical method to solve structural dynamic problems, whereas different discretization methods (e.g. Finite Differences and Finite Volumes) have been used in fluid mechanics problems. Now a days, many engineering fields are in need to merge (couple) these two approaches in order to tackle real-life engineering problems as physically correct as possible. When a plate vibrates in a fluid, the fluid offers resistance to the motion of the structure(plate). The resistance forces are inertial force and viscous force. Inertial force is due to acceleration of the plate whereas viscous force is proportional to velocity of the plate.

In literature, finite element formulation uses beam element or plate element for structural modelling which has a limitation on transverse displacement. Atkinson and Manrique [1] studied fluid reaction on a wide rectangular cantilever plate vibrating in a viscous fluid is constructed by superposition of a potential flow solution and an asymptotic correction built from the Navier–Stokes equations for an incompressible fluid with nonlinear terms neglected. Using the Wiener–Hopf method, a simple analytic expression for the fluid reaction is obtained for high values of the dimensionless parameter b, which is the frequency times the length of the plate squared divided by the kinematic viscosity. Fluid and structure motion are then coupled when solving the equation of motion of the plate with a fluid reaction expressed in terms of weighted averages of the velocity of the plate. The frequency response is then calculated for plates of different sizes and materials and the results are shown for a gas and a liquid. Catherine et al.[2] studied the finite amplitude vibrations of a cantilever beam of rectangular cross section immersed in a viscous fluid under harmonic base excitation. Fluid–structure interactions are modelled through a complex hydrodynamic function that describes added mass and damping effects in response to moderately large oscillation amplitudes. The hydrodynamic function is identified from the analysis of the two-dimensional flow physics generated by a rigid rectangle undergoing harmonic oscillations in a quiescent fluid. Computational fluid dynamics issued to investigate the effects of three salient non- dimensional parameters on the flow physics and inform the formulation of a tractable expression for the hydrodynamic function. Nagaya & Takeuchi [3] studied the vibration of the bottom in an arbitrarily shaped cylindrical container filled with fluid. In comparison, the investigation on the vibration of rectangular plates in contact with fluid is very limited. A mixed displacement/pressure formulation is used by Lei Cheng et al.[4] and they made a coupling element to establish interaction. Many studies [5],[6] in this field uses pressure based formulation to implement fluid interaction. And an analytical Ritz method is developed by Cheung and Zhou[7] to study the vibratory characteristics of an elastic rectangular plate in contact with fluid on one side. Kochupillai et al.[9] proposed a new formulation, based on the semi-analytical finite element method for elastic shells conveying fluids. The structural equations are based on the shell element while the fluid model is based on velocity potential. Dynamic pressure acting on the walls is derived from Bernoulli’s equation. Imposing the requirement that the normal components of velocity of the solid and fluid be equal, introduces fluid–structure coupling. The proposed technique has been validated using results available in the literature. This study shows that instability occurs at a critical fluid velocity corresponding to the shell circumferential mode with the lowest natural frequency and this phenomenon is also independent of the type of structural boundary conditions imposed.

As mentioned, review of the literature reveals that in most of the studies finite element formulation uses beam element or plate element for structural modelling which has a limitation on transverse displacement. Very few authors attempted fully coupled three dimensional FEM modelling for fluid-structure interaction. In the present work the coupling condition is enforced by equating the normal velocity of the fluid and the structure at the interface. Normal pressure acting on the surface of the structure is taken as load on the structure. A fully coupled analysis of a vibrating rectangular plate bounded by fluid will be done with finite element modelling of the system. The plate is partially submerged in fluid. Also a transient analysis will be attempted on the fluid structure interaction problem of a cantilever vibrating inside a box containing fluid. In this case the structure part will be modelled using ANSYS and the fluid part using CFX. The load transfer between the two fields will be done at the interface, where the information will be shared between the two different mesh through interpolation. In order to do this, a particular index will be used to specify the interfaces. In ANSYS, the surface will be flagged by an interface number, whereas in CFX, the surfaces will be flagged by an interface name (FSIN).

II. Finite element formulation

A. *Structure*

The eight-node isoparametric curved quadratic shell element (Fig. 1) has been used in this formulation. This element has five degrees of freedom (DOF) at each node, *u, v, w, α* and *β* where *u*, *v* and *w* are displacements along the *x*, *y* and *z* axes respectively. The displacements at any point with the isoparametric coordinates *ξ* and *η* are related to the nodal dof as given below:

$$u = \sum_{i=1}^{8} N_i u_i \qquad v = \sum_{i=1}^{8} N_i v_i \qquad w = \sum_{i=1}^{8} N_i w_i$$

$$\alpha = \sum_{i=1}^{8} N_i \alpha_i \qquad \beta = \sum_{i=1}^{8} N_i \beta_i \tag{1}$$

Where N_i is the shape function as given below:

$$N_i = \frac{(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)}{4}, \text{ for } i=1,2,3,4$$

$$N_i = \frac{(1+\xi\xi_i)(1-\eta^2)}{4}, \text{ for } i = 5, 7$$

$$N_i = \frac{(1+\eta\eta_i)(1-\xi^2)}{4}, \text{ for } i=6,8 \tag{2}$$

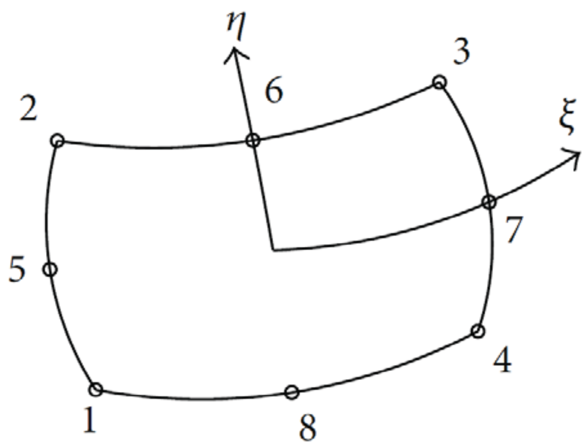

Figure 1: Eight node shell element

The generalized displacement vector of an element is expressed in terms of the shape functions and nodal degrees of freedom as

$$[u] = [N]\{d_e\} \tag{3}$$

That is,

$$\{u\} = \sum_{i=1}^{8} \begin{bmatrix} N_i & & & & \\ & N_i & & & \\ & & N_i & & \\ & & & N_i & \\ & & & & N_i \end{bmatrix} \begin{pmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{pmatrix} \tag{4}$$

The strain-displacement relation is given by:

$$[\varepsilon] = [B]\{d_e\} \tag{5}$$

Where

$$[B] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & -\frac{N_i}{R_y} & 0 & 0 \\ N_{i,y} & N_{i,x} & -\frac{2N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} \tag{6}$$

Strain energy is given by

$$U = 1/2 \int_v [\sigma][\varepsilon] dv \tag{7}$$

$$U = 1/2 \int_v [\varepsilon]^T [E][\varepsilon] dv \tag{8}$$

Substituting eq (5-6) on eq(8) and solving results the element stiffness matrix given by

$$[K_e] = \iint [B]^T [E][B] \, dx \, dy \tag{9}$$

The kinetic energy is given by,

$$T = 1/2 \int_v \rho(u^2 + v^2 + w^2) \, dv \tag{10}$$

Substituting eq(1-6) on eq(10) and solving results in the element mass matrix given by

$$[M_e] = \iint [N]^T [P][N] \, dx \, dy \tag{11}$$

Where

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & & & & \\ & N_i & & & \\ & & N_i & & \\ & & & N_i & \\ & & & & N_i \end{bmatrix} \quad \text{and}$$

$$[P]=\sum_{i=1}^{8}\begin{bmatrix}\rho & & & & \\ & \rho & & & \\ & & \rho & & \\ & & & \rho t^2 & \\ & & & & \rho t^2\end{bmatrix} \tag{12}$$

Where ρ is the mass density and t is thickness of structure.

B. Fluid domain

A twenty node, isoparametric brick element is used for modeling the fluid domain. The differential equation for the fluid region is governed by wave equation.

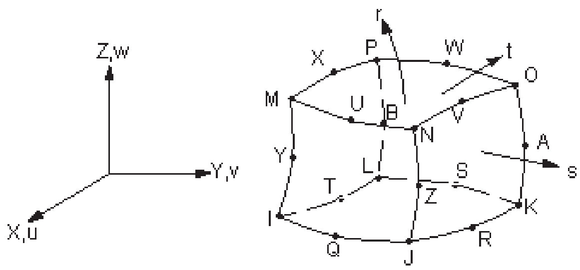

Figure 2: twenty node brick element

The following assumptions are made in deriving the equations, (1) the fluid flow is potential, (2) small deformation for structure, i.e, linear, (3) fluid is compressible and (4) there is no separation. Further, the velocity potential should satisfy the wave equation shown below:

$$\nabla^2\Phi - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U_x\frac{\partial}{\partial x}\right)^2 \Phi = 0 \tag{13}$$

Where ϕ is the velocity potential, c is the velocity of sound, U_x is the mean axial velocity of fluid (here U_x =0) and M=mach number.The velocity of fluid in the direction of plate is equal to instantaneous velocity of plate. This will satisfy the impermeability or dynamic boundary conditions, which ensures contact between the plate and fluid.

$$V_z = \frac{\partial\Phi}{\partial z} = \frac{\partial w}{\partial t} \tag{14}$$

A Galerkin weighted residual manipulation is used to formulate the finite element form of governing wave equation in cylindrical coordinates. The result of the manipulation is shown:

$$\int_v N_f^T\left(\nabla^2\Phi - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U_x\frac{\partial}{\partial x}\right)^2\Phi\right)dV = 0 \tag{15}$$

$$\int_s N_f^T\nabla\Phi.\mathbf{n}\,dS - \int_v N_f^T\,\nabla\Phi dV - \frac{1}{c^2}\int_v N_f^T\,\ddot{\Phi}dV - 2\frac{U_x}{c^2}\int_v N_f^T\frac{\partial^2\Phi}{\partial x\partial t}dV - \frac{U_x^2}{c^2}\int_v N_f^T\frac{\partial^2\Phi}{\partial x^2}dV = 0 \tag{16}$$

Neglect the coriolis and centrifugal components in the above equation. The first term of eq.(13) is rewritten using the fluid shell interface boundary condition as

$$\int_s N_f^T\,\nabla\Phi.\mathbf{n}\,dS = \int_s N_f^T\,\bar{N}ds\{\dot{U}_e\} + U_x\int N_f^T\frac{\partial\bar{N}}{\partial x}dS\{U_e\} \tag{17}$$

Where $\bar{N}$ is the *w* component of shell shape function and n is the unit normal vector to the structure.

Similarly the pressure acting on the fluid-structure interface can be converted to the finite element equations

$$\int_s \bar{N}^T\rho_f\left(\frac{\partial\Phi}{\partial t} + U_x\frac{\partial\Phi}{\partial x}\right)dS = \rho_z\int_s \bar{N}^T N_f dS\{\dot{\Phi}_e\} + \rho_f U_x\int_s \bar{N}^T\frac{\partial N_f}{\partial x}dS\{\Phi_e\} \tag{18}$$

Now the complete fluid-structure finite element equation is

$$\begin{bmatrix}M^{uu} & 0\\ 0 & G^{\Phi\Phi}\end{bmatrix}\begin{pmatrix}\ddot{u}\\ \ddot{\Phi}\end{pmatrix} + \begin{bmatrix}0 & C^{u\Phi}\\ -C^{\Phi u} & -U_xC^{\Phi\Phi}\end{bmatrix}\begin{pmatrix}\dot{u}\\ \dot{\Phi}\end{pmatrix} + \begin{bmatrix}K^{uu} & U_xK^{u\Phi}\\ -U_xK^{\Phi u} & H^{\Phi\Phi} - U_x^2I^{\Phi\Phi}\end{bmatrix}\begin{pmatrix}u\\ \Phi\end{pmatrix} = 0 \tag{19}$$

Rewritting the above equation

$$\lambda\begin{bmatrix}-C^* & -M^*\\ M^* & 0\end{bmatrix}\{d\} = \begin{bmatrix}K^* & 0\\ 0 & M^*\end{bmatrix}\{d\} \tag{20}$$

Where

$$M^* = \begin{bmatrix}M^{uu} & 0\\ 0 & G^{\Phi\Phi}\end{bmatrix},$$

$$K^* = \begin{bmatrix}K^{uu} & U_xK^{u\Phi}\\ -U_xK^{\Phi u} & H^{\Phi\Phi} - U_x^2I^{\Phi\Phi}\end{bmatrix},$$

$$C^* = \begin{bmatrix}0 & C^{u\Phi}\\ -C^{\Phi u} & U_xC^{\Phi\Phi}\end{bmatrix}$$

MATLAB code is written using above equations for a fully coupled fluid-structure interaction.

III. Coupled field analysis (ANSYS WORKBENCH)

A coupled field analysis is a multidisciplinary engineering analysis where various independent fields combine and interact together to solve a global engineering problem such that the result of one field is dependent on the other field(s). The coupling can be either one- way or two-way. In the one way coupling, the effect of one field is imposed on the other field but not the vice versa. In the fluid structure analysis, when the stiffness of the structure is too large, the deflection of such structure has a negligible impact on the flow field. Similarly, in the case of temperature-structure coupling, the temperature field affects the structural field by generating the thermal strains, but the structural strains have a negligible or no impact on the temperature field. For these types of applications, a one-way coupling analysis is sufficient. The classical approach towards one-way FSI is that the pressure distribution on the surface is calculated by CFD, which is exported to FEA to calculate the stresses and deflections on the structure.

A two-way coupling method is a more complex case where all the fields has a significant influence over each other. In the case of the fluid-structure analysis, when the deflection of the structure cannot be neglected, or in the case of the induction heating (magnetic-thermal analysis), two-way coupling strategy is essential. According to the ANSYS coupled- field guide, the coupled- field analysis is of two types: Sequential and Direct. The sequential method consists of two or more analyses of different fields, which are executed sequentially. The direct method on the other hand consists of only one analysis in which a coupled field element is used containing information from both the fields. Direct method is mostly used when the coupled-interaction is highly nonlinear. Sequential method offers independent solving of the different fields, providing more flexibility and efficiency when the coupled-interaction does not have a high degree of nonlinearity. Coupling can be sequentially done either by a physics file or by the multi-field solver (In the case of ANSYS, ANSYS-Multi field solver).

IV. 3D PLATE-FLUID CAVITY SYSTEM

A 3D rectangular acoustic cavity of size length 0:6m; breadth 0:5 m and thickness 0:4 m (see Fig. 3) completely filled with fluid (see table 1). One wall of the cavity is a flexible plate of thickness 6 mm clamped by its whole boundary. The other walls are considered perfectly rigid. The mechanical parameters of the plate are: density 7700 kg/m3, Young's modulus 1.44x1011 Pa and Poisson ratio 0:35. In the following section finite element analysis results, obtained from MATLAB code written using the above formulations, for the analysis of fluid-structure system shown in fig.3 is discussed. Firstly, a 3D free vibration elasto-acoustic problem analysed. Then, a comparison between the proposed approach and a full 3D analysis of same problem with results obtained from literatures conducted. Finally a comparison of first mode frequency of structural-acoustic system is shown with fluid inside the cavity changes.

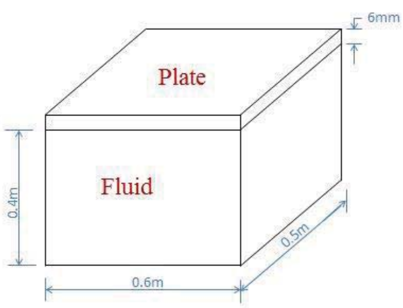

Fig. 3: Plate and Fluid cavity

TABLE 1: FLUIDS AND PROPERTIES

FLUIDS		
SUBSTANCE	***DENSITY(kg/m^3)***	***SONIC VELOCITY(m/s)***
Air	1	340
Kerosene	810	1324
Water	998	1450
Glycerol	1260	1904

V. RESULTS AND DISCUSSIONS

As mentioned earlier MATLAB program coded using displacement-potential formulation. A modal analysis of the system in which the cavity contains air is carried out using the program developed. The results of this modal analysis is compared with the results of Bermúdez et al. [8]. The comparison is given in table 2. below and it shows a very good match.

TABLE 2. FIRST MODE FREQUENCY

System	***Frequency (Hz)***	***Literature (Frequency(Hz))***
Structure	159.91	158.18
Fluid(Air)	281.86	283.85
FSI	158.01	158.18

A parametric study is conducted using different fluids (given in table 1) to determine the effects of density of the fluid with the frequency of fully coupled fluid-structure system. The point to be remembered is, here the analysis are carried out using potential formulation so the effects of viscosity of the fluid is neglected. The resulting first mode frequencies are shown below.

TABLE 3. FIRST MODE FREQUENCIES OF COUPLED SYSTEM

Fluids	***Coupled system frequency(Hz)***
Air	158.01
Kerosene	154.42
Water	151.16
Glycerol	138.82

The table 3 shows the coupled first mode frequency of the system is decreasing with increase of density as expected.

A. Transient analysis (fluid In-viscous)

A transient analysis of the same system without considering the effect of viscosity is conducted, transient force applied has got a magnitude of 1000N and frequency 900rad/s.

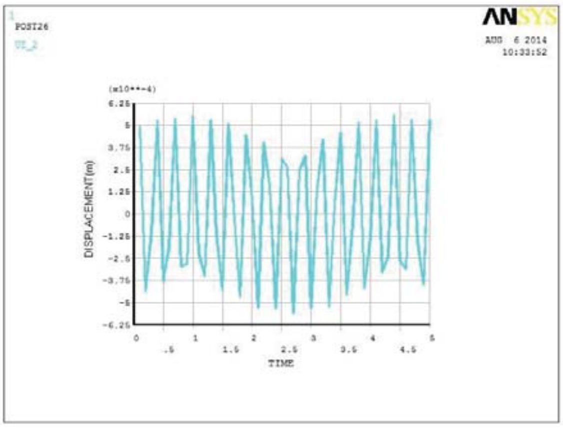

Fig.4. Transient analysis of the plate in air

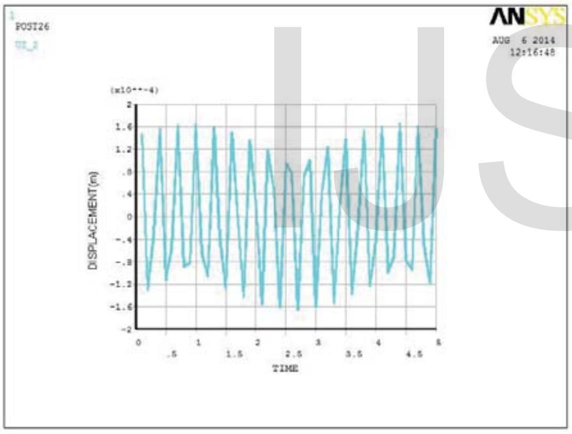

Fig.5. Transient analysis of the plate in water

The analysis is conducted both in air as well as in water. The graphs obtained are shown below. This analysis projects the reduction in amplitude of vibration when the fluid changed from air to water.

B. Transient analysis (viscous fluid)

A transient analysis of the same system is done using ANSYS WORKBENCH 14. Dimensions of the system used for the analysis is as follows Plate dimensions are 6mX5mX 0.006m. And dimensions of cavity is 6mX5mX4m. Dimensions are chosen to ensure that the plate generates a reasonable amplitude of vibration that doesn't decay too fast under the influence of fluid. An initial pressure of 100 Pa is applied to one side of the thin plate for 0.5 seconds in order to distort it. Once this pressure is released, the plate oscillates backwards and forwards as it attempts to regain its equilibrium (vertical) position. The surrounding fluid damps the plate oscillations, thereby decreasing the amplitude of oscillations with time. The CFX solver calculates how the fluid responds to the motion of the plate, and the ANSYS solver calculates how the plate deforms as a result of both the initial applied pressure and the pressure resulting from the presence of the fluid. Coupling between the two solvers is provided since the structural deformation affects the fluid solution, and the fluid solution affects the structural deformation. Here viscosity of the fluid also considered.

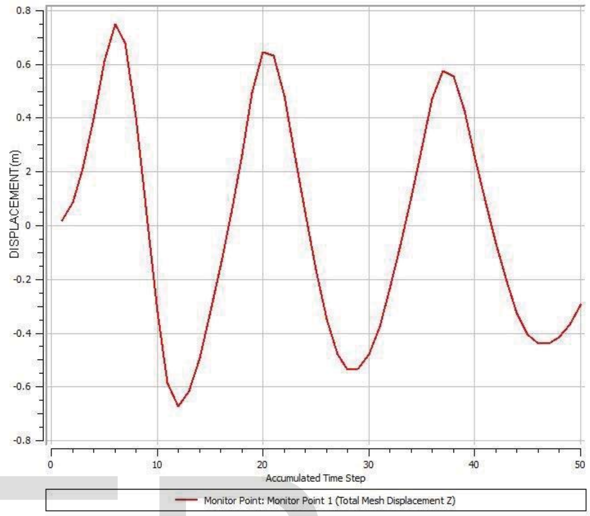

Fig.6.Transient analysis of the plate in air (viscosity 1.93x10^{-5})

From fig.6 and fig.7, it is understood that amplitude decay or damping of the plate vibration is more in water than the air. So it can be concluded that damping will be more with the fluid with high density and viscosity.

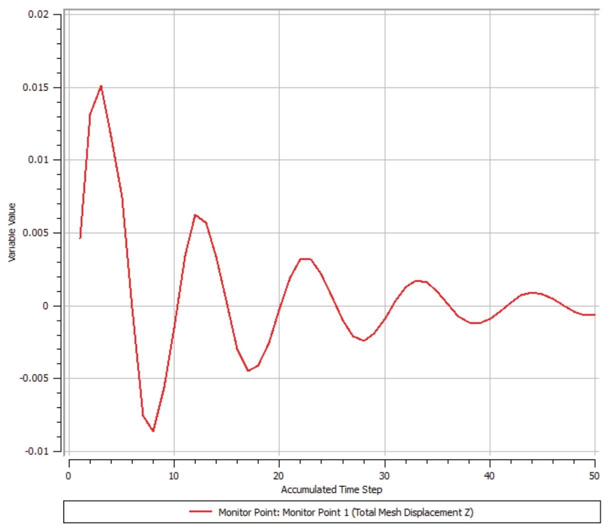

Fig.7.Transient analysis of the plate in water (viscosity 1x10^{-3})

Damping factor for air and water can be obtained using logarithmic decrement. From the fig 6. and Fig.7 the values of damping factor for air and water is obtained as 0.021 and 0.145 respectively.

VI. CONCLUSION

In this work, the dynamic response of a cantilever plate in air and partially submerged within different fluids has been studied. Using MATLAB code developed, modal analysis of plate coupled with different fluids conducted and results are shown in Table 3. Natural frequency of the plate is reduced with increase in density of the fluid. It is due to the added mass effect of the fluid acting on plate. A transient analysis using ANSYS CFX is carried out in which the viscosity of the fluid is incorporated for plate coupled with air and water. The results are shown in Fig.6 and Fig.7. Damping factor for air and water obtained as 0.0211 and 0.145 respectively

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